



# Highsted Grammar School

## A – Level

### Enrichment Booklet

*"We all use maths every day. To predict weather...to tell time...to handle money. Maths is more than formulas and equations. It's logic; it's rationality. It's using your mind to solve the biggest mysteries we know."*

# Introduction to Mathematics A-Level Enrichment at Highsted Grammar School.

Dear Students,

This booklet has been designed to take you beyond A – Level Maths and Further Maths into University level. Included in this booklet are questions from the Senior Team Maths Challenge, Oxford Admissions Test and STEP and AEA. At the end of this booklet, there is also a recommended reading list for you to have a look at to further extend your Mathematical knowledge.

I hope that you enjoy this and if you want to discuss any of the problems with your teachers then please do contact them.

Kind Regards,

**Mrs L Allen**

**Assistant Headteacher**

**Leader of Mathematics**

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Section 1: Senior Team Maths Challenge Questions

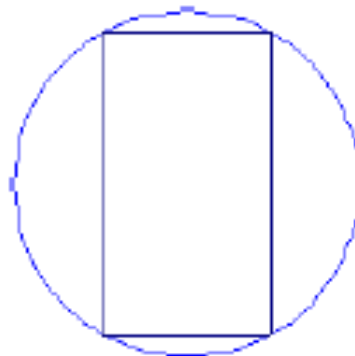
1 Given that  $y = (\sqrt{1+\sqrt{x}} + \sqrt{1-\sqrt{x}})(\sqrt{1+\sqrt{x}} - \sqrt{1-\sqrt{x}})$  find the value of  $y$  when  $x = 0.25$ .

2 The product

$$\sqrt{6} \times \sqrt{12} \times \sqrt{18} \times \sqrt{24} \times \sqrt{30} \times \sqrt{36} \times \sqrt{42}$$

can be expressed in the form  $2^a \times 3^b \times 5^c \times 7^d$ . What is the value of  $a+b+c+d$ ?

3 The circle in the diagram has radius 6 cm. If the perimeter of the rectangle is 28 cm, what is its area?



4 Let

$$ab = 104030$$

$$bc = 304010$$

$$a + c = 808.$$

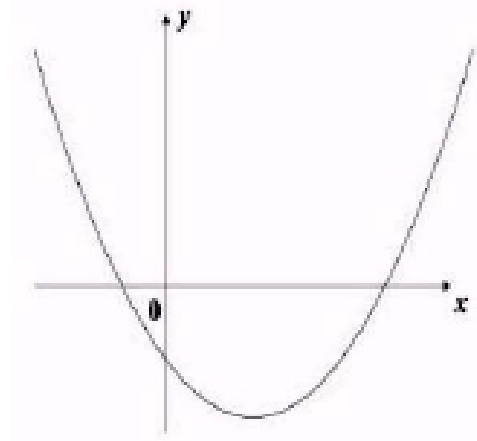
What is the value of  $b$ ?

5 How many two digit numbers  $N$  have the property that the sum of  $N$  and the number formed by reversing the digits of  $N$  is a square?

Section 2: Oxford Admissions Test Questions

1 (a) The diagram shows the graph of the function  $y = ax^2 + bx + c$ . Then:

- (i)  $b^2 - 4ac > 0$ ;      (ii)  $b^2 - 4ac = 0$ ;  
 (iii)  $b^2 - 4ac \leq 0$ ;      (iv)  $b^2 - 4ac < 0$ .



(b) The inequality  $2^n > n^2$  is true for:  
 (i) no integers  $n \geq 0$ ;      (ii) all integers  $n \geq 0$ ;      (iii) all integers  $n > 4$ ;  
 (iv) all integers  $n \geq 4$ .

(c) The simultaneous equations

$$ax + by = 1$$

$$cx + dy = 0$$

in  $x$  and  $y$ :

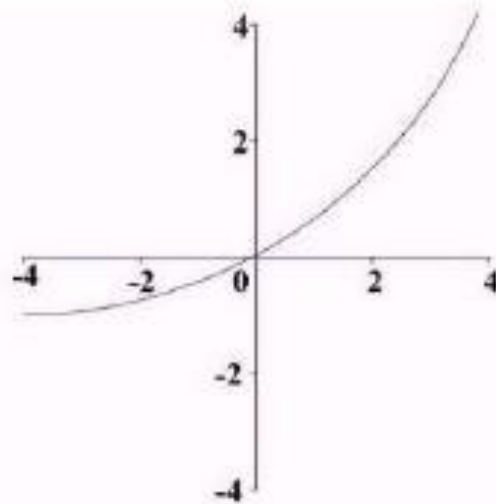
- (i) have a solution whatever the values of  $a, b, c, d$  may be;  
 (ii) have a unique solution whatever the values of  $a, b, c, d$  may be;  
 (iii) have a solution only if  $ad \neq bc$ ;  
 (iv) have a unique solution only if  $ad \neq bc$ .

(d) The complete set of solutions of the equation  $\sin 2x = \cos x$  in the range  $0 \leq x \leq 2\pi$  is:

- (i)  $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ ;      (ii)  $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$ ;      (iii)  $\left\{\frac{\pi}{6}, \frac{\pi}{2}\right\}$ ;      (iv)  $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ .

(e) if  $|x - 3| < 1$  and  $|x - 1| < 2$ , then:

- (i)  $-1 < x < 4$ ;      (ii)  $3 < x < 4$ ;      (iii)  $2 < x < 3$ ;      (iv)  $2 < x < 4$ .



(f) The diagram above shows the graph of the function  $y = f(x)$ . The graph of the function  $y = -f(x+1)$  is:



(g) As  $n$  becomes very large and positive,  $10000^{-\frac{1}{n}}$  approaches:

- (i) 0; (ii) 1; (iii) 10000; (iv)  $\infty$ .

(h) The derivative of the function  $y = (e^{\cos(5x)})^2$  is:

- (i)  $-5 \sin(5x)(e^{\cos(5x)})^2$ ; (ii)  $-20 \sin(5x) \cos(5x)(e^{\cos(5x)})^2$ ;  
 (iii)  $-10 \sin(5x)(e^{\cos(5x)})^2$ ; (iv)  $-10 \sin(5x) \cos(5x)(e^{\cos(5x)})^2$ . 4

(j) The derivative of the function

$$F(x) = \int_0^x f(t) dt$$

is:

(i)  $f(x) - f(0)$ ; (ii)  $f'(x)$ ; (iii)  $f(x)$ ; (iv)  $f'(x) - f'(0)$ .

(k) An entrance candidate is dealt three cards from a pack of fifty-two playing cards. To one significant figure the probability that he receives exactly one king is:

(i) 0.003; (ii) 0.01; (iii) 0.2; (iv) 0.05.

2. (a) Factorise the expression  $x^2 + x - 6$ .

(b) For which values of the real constant  $a$  does the equation

$$x^2 + x - a = 0$$

have at least one real solution? Write down these solutions in terms of  $a$ .

(c) Show that, for any value of the real constant  $b$ , the equation

$$x^3 - (b+1)x + b = 0$$

has  $x = 1$  as a solution. Find all values of  $b$  for which this equation has exactly two distinct solutions.

3. (a) Write down the equation of the straight line through the point (1,2) with slope -1.

(b) Let  $l$  be a line with equation

$$y = (2 - a) + ax,$$

where  $a$  is a constant. Show that, for any  $a$ , the line passes through the point (1,2). Find the equation of the line perpendicular to this line which also passes through the point (1,2).

(c) Find the equations of the lines which pass through the point (1,2) and have perpendicular distance 1 from the origin.

4. (a) Find the values of

1.  $\int_{-1}^1 (x^2 - x) dx$ ,

2.  $\int_{-1}^1 (x^3 + x^2 - 2x) dx$ .

(b) Sketch the graph of  $y = x^2 - x$  and indicate which difference in areas is represented by your answer to (a)(i).

(c) Find the total area (measured positively) that lies between the graphs of  $y = x^2 - x$  and  $y = x^3 + x^2 - 2x$  between  $x = -1$  and  $x = 1$ .

(d) The answers to (a)(i) and (a)(ii) are related in a particular way. Explain how the relationship can be seen *without* working out any integrals.

5. A total of 12 noughts and 4 crosses are arranged in 4 rows of 4. One such arrangement is illustrated below.

0	0	×	0
0	×	0	×
0	0	0	0
×	0	0	0

(a) How many arrangements are there altogether?

(b) How many arrangements are there in which there is a cross in every row?

(c) How many arrangements are there in which there is a cross in every row and in every column?

**Section 3: STEP & AEA Questions**

- 1** A pyramid stands on horizontal ground. Its base is an equilateral triangle with sides of length  $a$ , the other three sides of the pyramid are of length  $b$  and its volume is  $V$ .

Given that the formula for the volume of any pyramid is  $\frac{1}{3} \times$  area of the base  $\times$  height, show that

$$V = \frac{1}{12} a^2 (3b^2 - a^2)^{\frac{1}{2}}$$

The pyramid is then placed so that a non-equilateral face lies on the ground. Show that the new height,  $h$ , of the pyramid is given by

$$h^2 = \frac{a^2 (3b^2 - a^2)}{4b^2 - a^2}$$

Find, in terms of  $a$  and  $b$ , the angle between the equilateral triangle and the horizontal.

- 2** (i) The numbers  $m$  and  $n$  satisfy

$$m^3 = n^3 + n^2 + 1 \quad (*)$$

(a) Show that  $m > n$ . Show also that  $m < n + 1$  if and only if  $2n^2 + 3n > 0$ . Deduce that  $n < m < n + 1$  unless  $-\frac{3}{2} \leq n \leq 0$ .

(b) Hence show that the only solutions of (\*) for which both  $m$  and  $n$  are integers are  $(m, n) = (1, 0)$  and  $(m, n) = (1, -1)$ .

- (ii) Find all integer solutions of

$$p^3 = q^3 + 2q^2 - 1$$



- 3 (a) On the same diagram, sketch  $y = x$  and  $y = \sqrt{x}$ , for  $x \geq 0$ , and mark clearly the coordinates of the points of intersection of the two graphs.
- (b) With reference to your sketch, explain why there exists a value  $a$  of  $x$  ( $a > 1$ ) such that

$$\int_0^a x \, dx = \int_0^a \sqrt{x} \, dx.$$

- (c) Find the exact value of  $a$ .
- (d) Hence, or otherwise, find a non-constant function  $f(x)$  and a constant  $b$  ( $b \neq 0$ ) such that

$$\int_{-b}^b f(x) \, dx = \int_{-b}^b \sqrt{f(x)} \, dx.$$

## KS5 Reading List

### Websites:

**Plus Magazine** – Online general interest articles relating to practical uses of mathematics <https://plus.maths.org/content/>

**Nrich** - a selection of rich tasks for developing subject content knowledge as well as mathematical thinking and problem-solving skills. <https://nrich.maths.org/9088>

**Integral** - resources to support A Level maths and further maths courses. <https://integralmaths.org/> (please speak to your maths teacher for username and password).

### STEP Preparation:

**STEP support programme** by the University of Cambridge to help students develop problem-solving skills and prepare for university entrance exams: <https://maths.org/step/>

**STEP questions with solutions:**

<https://undergroundmathematics.org/browse?typeOption=ReviewQuestion&reviewQuestionType=6>

**Ebook: Advanced Problems in Mathematics** by Stephen Siklos (ISBN: 9781783747764) - this book bridges the gap between school and university mathematics and prepares students for an undergraduate mathematics course. The questions analysed in this book are all based on past STEP questions and each question is followed by a comment and a full solution.

<https://www.openbookpublishers.com/reader/1050#page/1/mode/2up>

### Books

**How to Study for a Mathematics Degree** by Lara Alcock (ISBN: 9780199661329) - this book explains what to expect at university

**Mathematics: A Very Short Introduction** by Timothy Gowers (ISBN: 9780192853615) - this book gives an idea of the scope and spirit of mathematics but is written in an accessible style.

**Number: A Very Short Introduction** by Peter M. Higgins (ISBN: 9780199584055) - this book unravels the world of numbers, demonstrating its richness

**Seventeen Equations that Changed the World** by Ian Stewart (ISBN: 9781846685316)

**Concepts of Modern Mathematics** by Ian Stewart (ISBN: 8601400596593)

**Fermat's Last Theorem** by Simon Singh (ISBN: 9781841157917)

**Mathematics: From the Birth of Numbers** by Jan Gullberg, Peter Hilton (ISBN: 9780393040029)

**The Man Who Knew Infinity: Life of the Genius Ramanujan New Ed** by Robert Kanigel (ISBN: 9780349104522)

**A History of Mathematics, Third Edition 3rd Revised edition** by Carl B. Boyer, Uta C. Merzbach (ISBN: 8601405590862)

**The Math Book: From Pythagoras to the 57th Dimension, 250 Milestones in the History of Mathematics** by Clifford A. Pickover (ISBN: 9781402788291)

**The Mathematics of Ciphers: Number Theory and RSA Cryptography** by S.C. Coutinho (ISBN: 9781568810829)

**Things to Make and Do in the Fourth Dimension** by Matt Parker (ISBN: 9780141975863)

### Further reading lists

University of Cambridge recommended reading list <https://www.maths.cam.ac.uk/documents/reading-list.pdf/>

Nrich reading list: <https://nrich.maths.org/9477>